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FIELD WIND WEIGHTING AND IMPACT PREDICTION FOR UNGUIDED ROCKETS

by Keith E. Hennigh

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SUMMARY

The mathematics for predicting impacts of unguided sounding rockets is presented. Wind measurements from several sources are combined with rocket wind response functions into equations suitable for digital computer application. This method has been used on four missile ranges, each utilizing a different type of digital computer.

CONTENTS

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INTRODUCTION

Unguided sounding rocket impact prediction is, essentially, the solution of the equations of motion. The limited knowledge of the rocket parameters, the uncertainty of wind velocities to be encountered, and the general complexity of the equations mitigate the accuracy of any achievable solution.

The ideal method of impact prediction for an unguided rocket would be to use a high speed electronic computer to solve the six-degree-of-freedom equations of the rocket in the presence of measured winds. But knowledge of the rocket parameters is usually not sufficient to warrant such a rigorous solution, and a computer program of this complexity could require as much as one minute of computation time per second of flight. These difficulties justify the adoption of a simplified field system such as the one presented here.

This system has been developed and used over a period of 3 years on four missile ranges for a large variety of unguided rockets. The computational portion of the system has been programmed for the IBM 709, IBM 650, Burroughs 220, and Bendix G-15 computers.

The purpose of this report is to present the mathematics used to translate wind measurements and rocket response functions into impact predictions. Application is limited to rockets fired "near vertical" as defined on page 3. The mechanics of implementing the model for a specific computer have been omitted in the interest of eliminating detail.

PRELIMINARY CONSIDERATIONS

A field facility must be capable of performing impact calculations for a variety of rockets without change of the basic system. The parameters used to describe the rocket response to the wind must be derivable both for rockets about which a minimum of information is known, and those about which full knowledge is available. Primarily, field operations are concerned with (1) wind velocity measurements as near flight time as possible and (2) the application of these measurements to impact prediction with the best available knowledge of the rocket description.

*This paper was prepared for Goddard Space Flight Center under Contract NAS 5-1032.

In this paper a set of rocket functions are defined which are independent of the variable knowledge of the rocket system. Three wind weighting functions that satisfy this independence have been adopted and are defined in the next section.

WIND WEIGHTING CONCEPT

The wind response of a rocket is described by three wind weighting functions. Numerical "firing" tables of these functions can be prepared in advance for each rocket, thereby removing the restrictions of limited computational time and mathematical complexity. Mathematical models can be generated which reflect the various degrees of available rocket description. Appendix A discusses current techniques.

Use of the selected wind weighting functions implies the assumption that the response of the rocket is linear with wind velocity, a necessity if precomputing the "firing" tables is to be permissible. Within this assumption, the functions defined and discussed below describe the wind response characteristics of the rocket.

As a basis for the definitions of the wind weighting functions, it is desirable first to define the "ballistic wind" velocity. The ballistic wind velocity W is a hypothetical wind which is constant in direction and magnitude from the ground level to a defined upper limit of the effective atmosphere z_{max} .* The integrated effect of W on the rocket impact is equal to the integrated effect of the actual wind.

The wind weighting function $f(z)$ is the integral of the weighting function df/dz. By defining $w(z)$ to be the actual wind velocity the equation

$$W = \int_0^{z_{max}} \frac{df}{dz} w(z)\, dz$$

$$= \sum_i \Delta f_i \, w\left(z_i\right) \quad , \tag{1}$$

becomes the functional relationship between W, df/dz, and $w(z)$. Figures 1 and 2 are typical $f(z)$ functions. Preparation of wind weighting functions for a particular rocket involves computations for a series of initial launch angles, θ. The computations reveal $f(z)$ to be a function of θ; however, this variation of $f(z)$ is small and is neglected. The accepted procedure is to select the $f(z)$ function associated with the planned "effective Q.E."† If the actual effective Q.E. is within ±5 degrees of that planned, the use of the chosen $f(z)$ is considered valid.

*In this report the upper limit of the effective atmosphere is 100,000 ft above mean sea level. This value is selected since available measurement techniques are generally bounded by this region, and the class of rockets being considered can tolerate the lack of wind measurements above this altitude.

†"Effective Q.E." is defined on page 12.

The $f(z)$ values used are normally computed to an altitude of 100,000 ft with the intention of measuring winds up to this altitude. In practice it can be expected that winds will be measured to a nominal height of 80,000 ft.

The small percentage contribution of the higher regions exhibited in $f(z)$ has caused the common practice of ignoring high altitude wind measurements and considering their effects negligible. This practice is not always justified, since high wind velocities can cause the contributions to $\mathbf{W}$ from a small portion of $f(z)$ in higher regions to be quite large. In assessing the potential consequences of unknown upper winds, measurements and calculations for a specific activity are necessary. In the subsequent discussion $f(z)$ will be referred to as a set of tabular values $\{z_i, f_i\}$, subject to linear interpolation for intermediate values.

The unit wind effect $\delta(\theta)$ is the magnitude of the impact displacement vector due to a unit ballistic wind to the height z_{max}. The value θ is the launcher tilt angle. Under the assumption of linear response to the wind, the impact displacement vector due to the winds is $\delta(\theta)\mathbf{W}$. Figure 3 is a typical $\delta(\theta)$ function. The definition of $\delta(\theta)$ is extended to provide for cross- and range-wind effects. The definition in matrix notation is

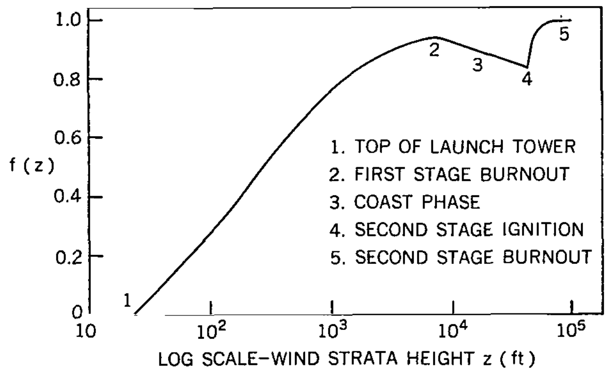

Figure 1—Typical $f(z)$ curve for a two stage configuration.

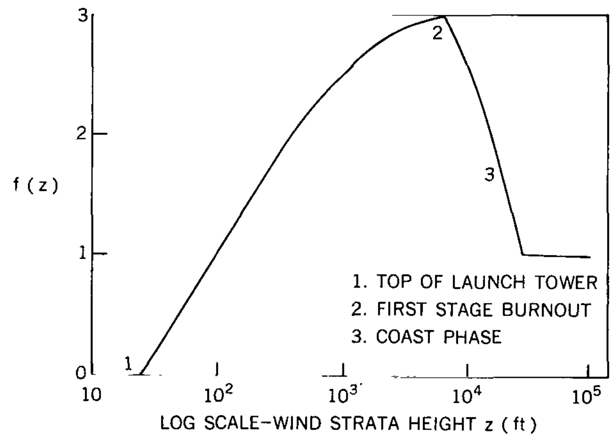

Figure 2—Typical $f(z)$ curve for a one stage configuration.

$$\delta(\theta) = \begin{pmatrix} \delta_c(\theta) & 0 \\ 0 & \delta_R(\theta) \end{pmatrix} , \tag{2}$$

where δ_R and δ_c are the range- and cross-wind effects in the direction of the tower azimuth and normal to it, respectively.*

$R(\theta)$ is the no-wind impact range of the rocket (Figure 4). It should be noted that the inverse function $\theta(R)$ is a multiple-valued function. By restricting the solution to the value nearest the vertical (hereafter referred to as the "near vertical" solution) the ambiguity is removed. This restriction is valid since sounding rockets are launched to achieve maximum peak altitudes.

*The introduction of the cross- and range-wind effects was prompted by the belief that different effects could be computed. The capability of the operational scheme exists, but an independent evaluation of δ_c has not been attempted. In current application $\delta_c = \delta_R$.

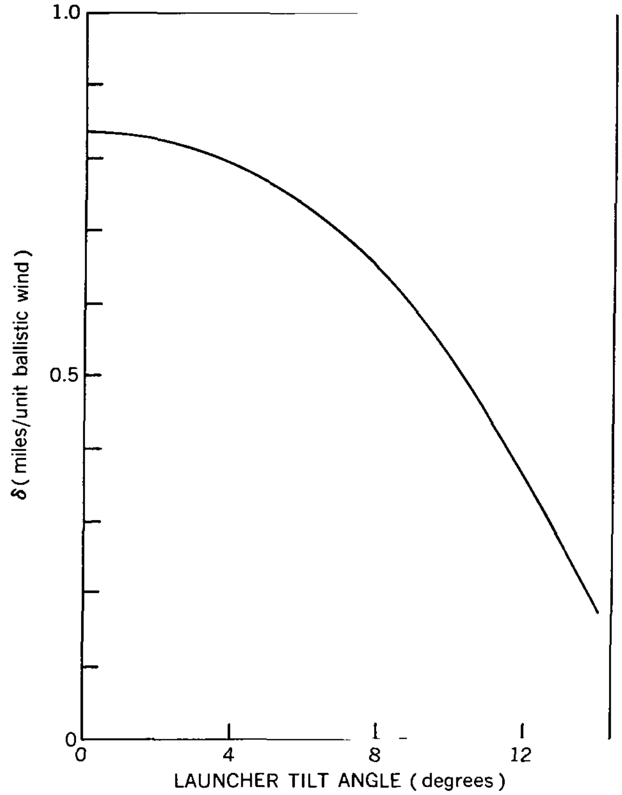

Figure 3—Typical $\delta(\theta)$ curve.

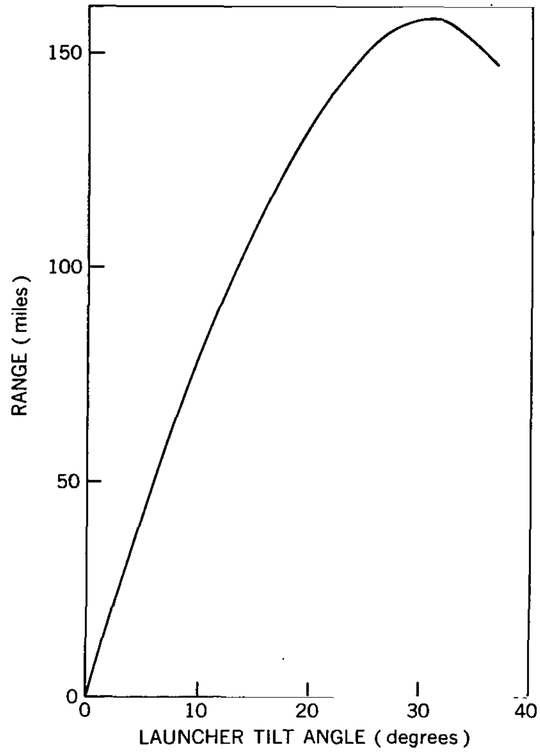

Figure 4—Typical no-wind impact range curve.

In subsequent discussions, the unit wind effect and no-wind range will be referred to as a set of tabular values $\{\theta_i, R_i, \delta_{ci}, \delta_{Ri}\}$. The tabular values will be such that either θ or R may be arguments and linear interpolation is acceptable for intermediate values.

For the near vertical firings, it is assumed that the effects of the earth's rotation will be independent of the flight path. A constant coriolis vector, $C(C_x, C_y)$ is predetermined (Appendix A, page 19).

NOMENCLATURE

All measurements are treated in a right-hand Cartesian coordinate system with x, y, z being positive east, north, and vertical, respectively. Azimuth angles are measured clockwise from north and elevation angles from the local xy plane. The term "tilt" (or "θ") refers to the angle between z and the launcher. Quadrant elevation (Q.E.) is the complement of the tilt angle.

The indexing system for all variables will be such that

$$\Delta\xi_i = \xi_i - \xi_{i-1}, \text{ where } i = 1, 2, 3, \cdots, n,$$

and

$$\xi_i = \sum_{i=1}^{n} \Delta\xi_i ,$$

where $\Delta\xi_0 = 0$, by definition. This convention establishes first forward differences and associates values, such as velocities, with the upper boundary of the interval of computation.

DUTIES OF A FIELD FACILITY

The field duties for impact prediction may be divided conveniently into two categories, evaluation of the ballistic wind and evaluation of the launcher tilt and azimuth or the impact vector.

In ballistic wind evaluation, it is desirable to utilize every available type of wind sensing device, combining the results into a composite wind profile to the maximum attainable altitude. The ballistic wind is obtained from this profile by means of Equation 1. Sources used to measure wind velocity are:

1. Rawinsonde to high altitudes;
2. Radar tracking of balloons ascending to high and intermediate altitudes;
3. Double theodolite optical tracking of balloons ascending to relatively low altitudes;
4. Anemometer measurements of surface winds.

All or part of the above are combined to provide a wind velocity profile.

Figure 5 demonstrates the necessity for combining wind measurements from several sources into a single profile. This chart depicts a typical balloon tracking schedule in terms of altitude

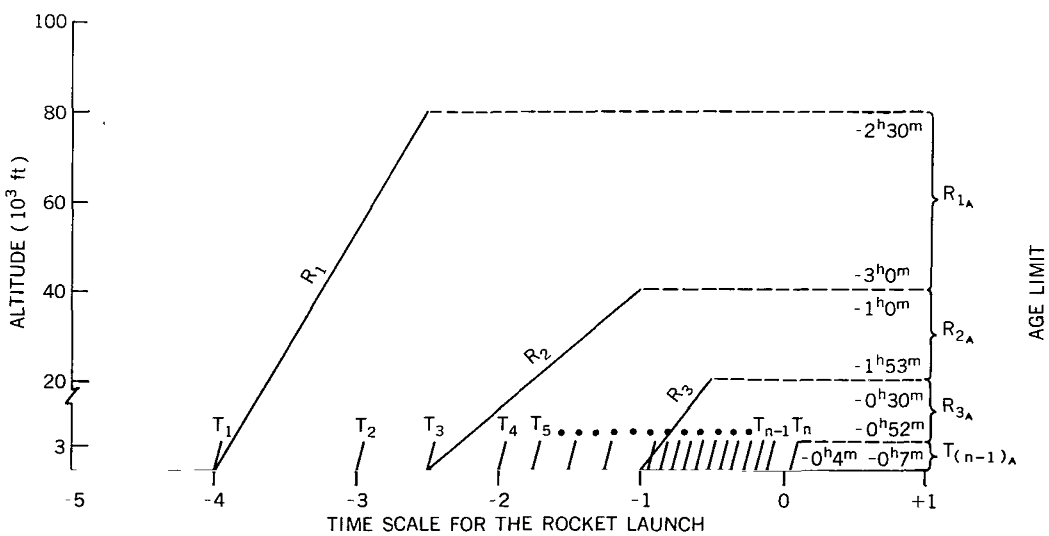

Figure 5—Typical prelaunch wind measurement schedule.

vs. time relative to the anticipated time of rocket launch. R_1, R_2, and R_3 represent radar-tracked balloons and $T_1, T_2, \ldots, T_{n-1}, T_n$, a series of theodolite-tracked balloons. An examination of the figure reveals the construction given in Table 1 for a wind profile, if the most current measurements in each altitude region are considered. The values show the earliest wind measurements to be 3 hours old and each lower adjacent stratum to be more current. This pattern becomes more meaningful when the effects of the various wind strata on the response of a rocket are considered. Response to lower level winds is greater than the response to higher level winds. Furthermore, higher level winds are, in general, more stable and measurements of them will remain valid for relatively longer times.

During an operation, comparison of R_2 with R_1 in the regions of z-overlap can build confidence in the remaining portion of R_1, or might show the need to continue the R_2 measurement to higher altitudes and eliminate the R_1 values. The multiple theodolite releases indicated in Figure 5 and Table 1 provide information pertaining to the stability of the lower winds. However, it is not the purpose of this paper to provide "ground rules" for evaluating the data; the intention is to develop the techniques for rapidly combining them. It is believed that Figure 5 and the foregoing discussion demonstrate the adopted concept of rapid "up-dating" of data.

Table 1

A Wind Profile Constructed from Several Balloon Measurements.

Source	Altitude Coverage (10^3 ft)	Age Limit	Average Age
R_1	$40 \le z \le 80$	$-3^h\,0^m \le t \le -2^h\,30^m$	$2^h\,45^m$
R_2	$20 \le z \le 40$	$-1^h\,53^m \le t \le -1^h\,0^m$	$1^h\,26.5^m$
R_3	$3 \le z \le 20$	$-0^h\,52^m \le t \le -0^h\,30^m$	$0^h\,41^m$
T_{n-1}	$0 \le z \le 3$	$-0^h\,7^m \le t \le -0^h\,4^m$	$0^h\,5.5^m$

WIND VELOCITY DETERMINATION

The four wind sensing devices may be divided into two groups from the viewpoint of velocity determination:

1. Radar and theodolite tracking require the resolution of measured values into space position and wind velocities;
2. Rawindsonde and anemometers require the conversion of measured velocities into proper components and units.

In all cases it is desired to transform the input data sets of the form $\{z_i, w_{x_i}, w_{y_i}\}$, where z_i is the top level of the altitude stratum, w_{x_i} and w_{y_i} are the average horizontal wind velocity components bounded by the interval, and $z_{i-1} < z \le z_i$. Equations required to transform the four types of wind measurements into this general form will be discussed below. The evaluation of the ballistic wind components and methods for combining the various segments will then be developed.

Radar

Balloon position values may be taken directly from the radar plot board in the convenient form $\{t_i, x_i', y_i', z_i'\}$, where t_i is the time and the primes denote values not possessing a necessary parallax correction. The equations

$$\left.\begin{aligned} x_i &= x_i' + p_x \,, \\ y_i &= y_i' + p_y \,, \\ z_i &= z_i' + p_z \,, \end{aligned}\right\} \tag{3}$$

transform the original data into the form $\{t_i, x_i, y_i, z_i\}$. Equations for velocity determination are listed later.

The required parallax correction results from an operational necessity to adjust the radar plot board parallax, scale factors, or both during a tracking mission. Prior to the balloon release, the plot board is set up with the desired scale factors, and parallax is nulled to zero. During the mission the necessity of performing a plot board adjustment may occur several times. Such an adjustment requires a rapid change of the plot board parallax, scale factors, or both, thus interrupting the continuity of the data plot. The assumption is that the velocity remains constant during the brief interruption and the next space point may be evaluated through extrapolation.

In the following equations superscripts (1) and (2) denote observations made before and after the parallax adjustment, respectively.

The extrapolated i^{th} values are

$$\left.\begin{aligned} x_i^{(1)} &= x_{i-1}^{(1)} - \dot{x}_{i-1}^{(1)} \left(t_i - t_{i-1} \right) \,, \\ y_i^{(1)} &= y_{i-1}^{(1)} - \dot{y}_{i-1}^{(1)} \left(t_i - t_{i-1} \right) \,, \\ z_i^{(1)} &= z_{i-1}^{(1)} - \dot{z}_{i-1}^{(1)} \left(t_i - t_{i-1} \right) \,, \end{aligned}\right\} \tag{4}$$

and the parallax correction is

$$\left.\begin{aligned} p_x &= x_i^{(1)} - x_i^{(2)} \,, \\ p_y &= y_i^{(1)} - y_i^{(2)} \,, \\ p_z &= z_i^{(1)} - z_i^{(2)} \,. \end{aligned}\right\} \tag{5}$$

The negative signs of Equations 4 allow the utilization of velocity components previously computed. The only effect of the parallax correction to the wind velocity profile is that the velocity remains constant over the interval $z_{i-1} \leq z \leq z_i$. The adjusted space positions can be used for comparison purposes.

If any entry in the data set $\{t_i, x_i', y_i', z_i'\}$ follows a parallax adjustment, the new parallax is computed before the application of Equations 3.

Theodolite

Figure 6 represents a double theodolite system having its baseline in a local horizontal plane. Observations are of the form $\left\{t_i, A_{1_i}, E_{1_i}, A_{2_i}, E_{2_i}\right\}$, where t_i is the time of the observation; A_1 and E_1 are the azimuth angle measured from the north and the elevation angle measured from the horizontal plane by theodolite station 1; and A_2 and E_2 are the azimuth angle measured from the north and the elevation angle measured from the horizontal plane by theodolite station 2. A general solution of the space position with respect to theodolite station 1 is given by:

$$\left.\begin{aligned}
\phi_1 &= \cos^{-1}\left[\cos E_1 \cos\left(A_1 - \alpha\right)\right] , \qquad \left(0 < \phi_1 < \pi\right) , \\
\phi_2 &= \cos^{-1}\left\{\cos E_2 \cos\left[\pi - \left(A_2 - \alpha\right)\right]\right\} , \qquad \left(0 < \phi_2 < \pi\right) , \\
\gamma &= \pi - \left(\phi_1 + \phi_2\right) , \qquad \left(0 < \gamma < \pi\right) , \\
S_1 &= \frac{B \sin\phi_2}{\sin\gamma} , \\
z_i &= S_1 \sin E_1 , \\
x_i &= S_1 \cos E_1 \sin A_1 , \\
y_i &= S_1 \cos E_1 \cos A_1 ,
\end{aligned}\right\} \qquad (6)$$

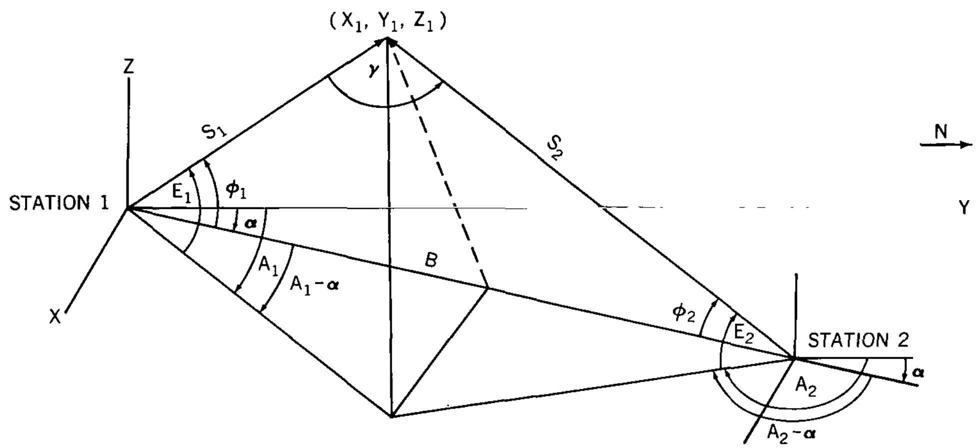

Figure 6—Double theodolite geometry.

where α is the angle between north and the baseline measured at station 1, and B is the length of the baseline. This solution is invalid only when $\gamma = 0$ or π. Velocity is determined by the equations which follow.

Velocity Equations

For both radar and theodolite the velocity equations are:

$$\left.\begin{aligned} w_{x_i} &= \frac{x_i - x_{i-1}}{\Delta t} , \\ w_{y_i} &= \frac{y_i - y_{i-1}}{\Delta t} , \\ -\dot{z}_i &= \frac{z_i - z_{i-1}}{\Delta t} , \end{aligned}\right\} \qquad (7)$$

where $\Delta t = t_{i-1} - t_i$. The reversal of the order of time indices is intentional. It provides the components of a velocity vector which points into the wind, and in the direction of rocket response. The values w_{x_i} and w_{y_i} are used in deriving the ballistic wind; $\dot{z}_i$ is derived solely to be used for radar parallax computation if required.

Rawinsonde

Rawinsonde data is provided in a form $\{z_i, v_i, a_i\}$, where z_i is the top of a wind stratum bounded by $z_{i-1} < z \leq z_i$, v_i is the magnitude of the wind velocity in this region, and a_i is the direction of the wind velocity vector, indicating the direction in which the wind is blowing. Treatment of these data requires only the conversion to velocity components:

$$\left.\begin{aligned} w_{x_i} &= -k_1 v_i \sin a_i , \\ w_{y_i} &= -k_1 v_i \cos a_i , \\ z_i &= k_2 z_i , \end{aligned}\right\} \qquad (8)$$

where k_1 and k_2 are conversion constants required to convert units to ft/sec and feet, respectively. The negative signs give the reverse direction to that of the wind vector, as mentioned in the previous section.

Anemometer

Experiments involving the incorporation of anemometer data into impact prediction calculations have been included in the Burroughs 220 program. The possibility of using these data for

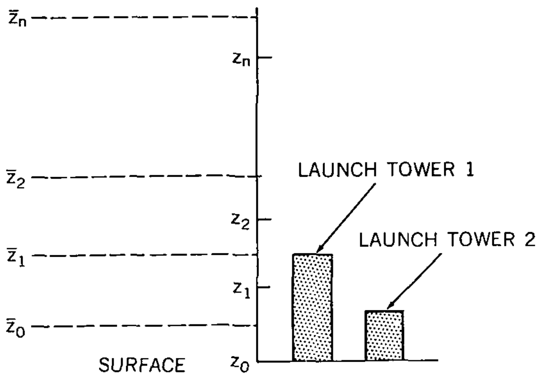

Figure 7—Diagram of the anemometer system.

rapid updating of the surface wind effect on the impact prediction has not been completely evaluated. Operations involving the use of this feature have been hampered by the slow and cumbersome manual input of data, and by the limitations of the existing facilities for measuring the altitude stratum.

The technique developed assumes a tower with a number of anemometers spaced at arbitrary, but known, heights, z_i, above the surface (Figure 7).

Each anemometer is capable of measuring wind velocity in terms of direction a_i and magnitude v_i, at a single point in space. In contract to wind measurements taken by observing sequential space positions of a balloon and averaging over a stratum, it is necessary to define a similar stratum over which the point velocity is to be applied. It is assumed that the wind remains constant over a region extending from the anemometer under consideration to an altitude one-half the distance to adjacent anemometers. In the case of the uppermost anemometer, the stratum is extrapolated to a height one-half the distance between the two uppermost anemometers.

The stratum boundaries, consistent with the adopted indexing system, can be stated:

$$\bar{z}_{i-1} = \frac{z_i + z_{i-1}}{2}, \text{ where } i = 1, 2, 3, \cdots, n-1,$$

and

$$\bar{z}_n = \frac{z_n - z_{n-1}}{2} + z_n .$$

Also,

$$w_{x_i} = kv_i \sin a_i$$

and

$$w_{y_i} = kv_i \cos a_i$$

are the equations required to evaluate the wind components; k is the required conversion factor. The above equations provide $\left\{z_i, w_{x_i}, w_{y_i}\right\}$ for a given anemometer array.

BALLISTIC WIND EVALUATION

To illustrate the method of combining two sets of data into a combined wind profile, we define the sets:

$$\{z_i, w_i\}, \text{ where } i = 1, 2, 3, \cdots, n, \qquad (0 \le z_i \le z_n),$$

and

$$\{z_j, w_j\}, \text{ where } j = 1, 2, 3, \cdots, m, \qquad (0 \le z_j \le z_m),$$

where $w_i \left(w_{x_i}, w_{y_i}\right)$ is the wind velocity bounded by $z_{i-1} < z \le z_i$. The j set is considered a more recent observation. All the $\{z_i, w_i\}$ elements bounded by $0 \le z_i \le z_m$ represent a stratum common to $\{z_j, w_j\}$. The j elements common to this stratum are to replace the i elements (the earlier set of data). The subset of $\{z_i, w_i\}$ for which $z_i > z_m$ is then

$$\{z_i', w_i'\}, \text{where } i' = 1, 2, 3, \cdots, m',$$

and the union of the i' and j sets represents a new set:

$$\{z_k, w_k\}, \text{ where } k = 1, 2, 3, \cdots, (m' + j), \ (0 \le z \le z_n).$$

Equation 1 applied to this new set yields

$$W = \sum_{k=1}^{m'+j} \Delta f_k w_k,$$

where W is the ballistic wind with the most recent information from the i and j observations. The combined k set can be treated as the i set when new data are introduced.

Methods and rules for performing an operation equivalent to the above in a digital computer are developed in Appendix B.

IMPACT AND TILT DETERMINATION

The effect of W on a rocket impact is presented below. The problem is phrased in two ways:

1. For a given launcher tilt θ and azimuth ϕ_R evaluate the predicted impact range I and azimuth ϕ_I;

2. For a given impact range and azimuth evaluate the required tower tilt and azimuth.

As a convenience the terms to be used are now listed: The ballistic wind vector is

$$W = \begin{pmatrix} W_x \\ W_y \end{pmatrix};$$

the constant Coriolis vector is

$$C = \begin{pmatrix} C_x \\ C_y \end{pmatrix};$$

the unit wind effect matrix is

$$\delta(\theta) = \begin{pmatrix} \delta_c(\theta) & 0 \\ 0 & \delta_R(\theta) \end{pmatrix} ,$$

and the tabular set defining the functions $R(\theta)$, $\delta_c(\theta)$, $\delta_R(\theta)$ is $\left\{\theta_i, R_i, \delta_{c_i}, \delta_{R_i}\right\}$.

The required equation for determining the impact range and azimuth for a given launch angle and azimuth (Figure 8) is:

$$\mathbf{I} = \mathbf{R}(\theta) + \mathbf{D}(\theta, \mathbf{W}) + \mathbf{C} .$$

In the reference frame oriented about $\mathbf{R}$ (or y') the individual vectors are:

$$\mathbf{R}(\theta) = \begin{pmatrix} 0 \\ R(\theta) \end{pmatrix} ,$$

$$\mathbf{D}(\theta, \mathbf{W}) = \delta(\theta)\, M \begin{pmatrix} W_x \\ W_y \end{pmatrix} ,$$

$$\mathbf{C} = M \begin{pmatrix} C_x \\ C_y \end{pmatrix} ,$$

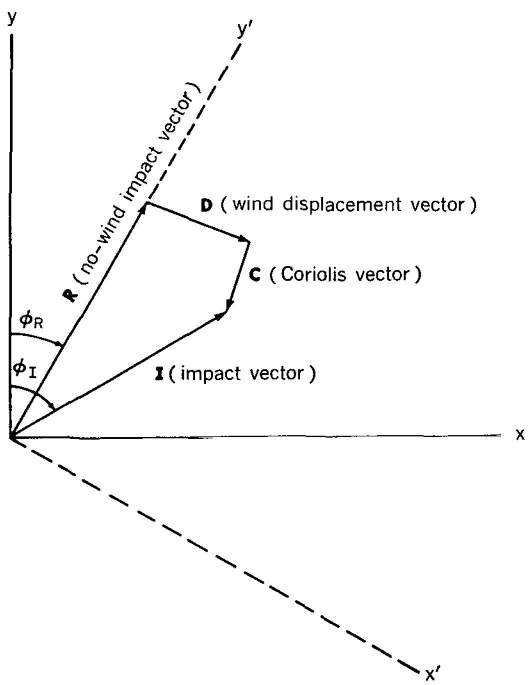

Figure 8—Impact geometry.

where

$$M = \begin{pmatrix} \cos\phi_R & -\sin\phi_R \\ \sin\phi_R & \cos\phi_R \end{pmatrix} .$$

In polar form, the required solution is

$$I = \sqrt{I_x^2 + I_y^2} , \qquad (9)$$

$$\phi_I = \phi_R + \tan^{-1}\frac{I_x}{I_y} . \qquad (10)$$

An auxiliary parameter introduced to give information concerned with the probable "pitch-up" of the rocket is the effective quadrant elevation (Q.E.) or effective tilt. It is equivalent to the tilt angle required to achieve the specified impact in the presence of no-wind. The value is obtained from the set $\left\{\theta_i, R_i, \delta_{c_i}, \delta_{R_i}\right\}$ by using I for the argument R.

Effective Q.E. appears to be a suitable criterion for determining the approximate trajectory path since experience and computer simulations indicate that a large percentage of the total wind response occurs very early in flight. The planned effective Q.E. determines the choice of the $f(z)$ function.

The required equation for determining launcher tilt and azimuth for a given impact range and azimuth (Figure 9) is:

$$\mathbf{R} = \mathbf{I} - \mathbf{C} - \mathbf{D}(\mathbf{R}, \mathbf{W}) .$$

$\mathbf{D}$ is a function of $\mathbf{R}$ and therefore requires an iteration process. A first approximation, $\mathbf{R}^{(1)}$, is evaluated in a reference frame about $\mathbf{I}$ (or y') in Figure 9. In the primed reference frame,

$$\mathbf{R}^{(1)} = \begin{pmatrix} R_x^{(1)} \\ R_y^{(1)} \end{pmatrix} ,$$

$$\mathbf{I} = \begin{pmatrix} 0 \\ I \end{pmatrix} ,$$

$$\mathbf{C} = B \begin{pmatrix} C_x \\ C_y \end{pmatrix} ,$$

$$\mathbf{D} = \delta(I)\, B \begin{pmatrix} W_x \\ W_y \end{pmatrix} ,$$

$$B = \begin{pmatrix} \cos\phi_I & -\sin\phi_I \\ \sin\phi_I & \cos\phi_I \end{pmatrix} ,$$

$$\delta(I) = \begin{pmatrix} \delta_c(I) & 0 \\ 0 & \delta_R(I) \end{pmatrix} .$$

Elements of $\delta(I)$ are selected from $\{\theta, R, \delta_c, \delta_R\}$, by using I as the argument. In polar form the solution is:

$$R^{(1)} = \sqrt{\left(R_x^{(1)}\right)^2 + \left(R_y^{(1)}\right)^2} ,$$

and

$$\alpha^{(1)} = \tan^{-1} \frac{R_x^{(1)}}{R_y^{(1)}} .$$

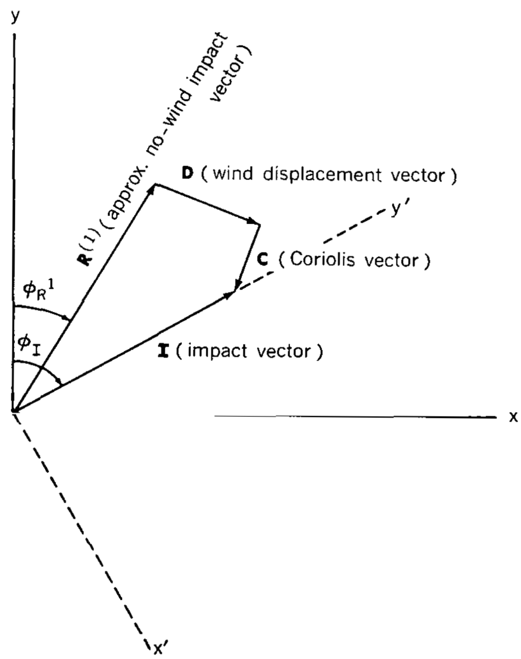

Figure 9—Tilt geometry, first approximation.

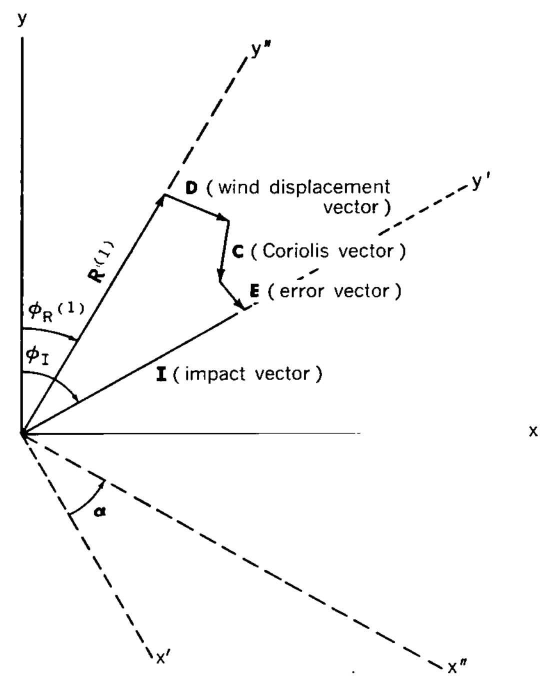

Figure 10—Tilt geometry, recursion relation.

If

$$A^{(1)} = \begin{pmatrix} \cos \alpha^{(1)} & -\sin \alpha^{(1)} \\ \sin \alpha^{(1)} & \cos \alpha^{(1)} \end{pmatrix} ,$$

the matrix product

$$M^{(1)} = AB$$

represents a rotation matrix through the angle $\phi_R^{(1)}$.

Figure 10 demonstrates the following recursion equation:

$$\mathbf{E} = \mathbf{I} - \mathbf{R}^{(1)} - \mathbf{D}^{(1)} - \mathbf{C} , \qquad (11)$$

where E is an error vector whose magnitude must be made smaller than some value ϵ. The equation is expressed in a reference oriented about $\mathbf{R}^{(1)}$ (or y″) in Figure 10 and

$$\mathbf{E} = \begin{pmatrix} E_x \\ E_y \end{pmatrix} ,$$

$$\mathbf{I} = A^{(1)} \begin{pmatrix} 0 \\ I \end{pmatrix} ,$$

$$\mathbf{R}^{(1)} = \begin{pmatrix} 0 \\ R^{(1)} \end{pmatrix} ,$$

$$\mathbf{D} = \delta \left(R^{(1)}\right) M^{(1)} \begin{pmatrix} W_x \\ W_y \end{pmatrix} ,$$

$$\mathbf{C} = M^{(1)} \begin{pmatrix} C_x \\ C_y \end{pmatrix} .$$

In polar form

$$\mathbf{E} = \sqrt{E_x^2 + E_y^2} .$$

If $E < \epsilon$, the problem is solved. Elements of $M^{(1)}$ are used to compute

$$\phi_R = \tan^{-1} \frac{\sin \phi_R^{(1)}}{\cos \phi_R^{(1)}} ;$$

the magnitude of the no-wind impact is $R^{(1)}$, and the launcher tilt is the value of θ from $\{\theta_i, R_i, \delta_c, \delta_R\}$ with $R^{(1)}$ as the argument. If $E \geq \epsilon$, a correction must be applied to $R^{(1)}$:

$$R^{(2)} = |\mathbf{R}^{(1)} - \mathbf{E}| \cdot$$

The angle between $\mathbf{R}^{(1)}$ and $\mathbf{R}^{(2)}$ may be determined by

$$\alpha^{(2)} = \tan^{-1} \frac{R_x^{(2)}}{R_y^{(2)}} \cdot$$

The matrix

$$M^{(2)} = A^{(2)} M^{(1)} ,$$

where

$$A^{(2)} = \begin{pmatrix} \cos \alpha^{(2)} & -\sin \alpha^{(2)} \\ \sin \alpha^{(2)} & \cos \alpha^{(2)} \end{pmatrix} ,$$

is the required rotation matrix.

After setting

$$A^{(1)} = A^{(2)} ,$$

$$M^{(1)} = M^{(2)} ,$$

$$R^{(1)} = R^{(2)} ,$$

the process is repeated beginning with Equation 11.

CONCLUSION

The procedures given in this report do not depart from accepted theories and methods for measuring wind velocity profiles and applying these wind profiles to rocket impact prediction.

The purpose of this work was to provide a systematic formulation of these methods into a set of equations adaptable to digital computers.

Application of the process to field operations will provide uniform measurements forming a collection of data useful in:

1. Demonstrating the validity of, or improving methods for, impact prediction;

2. Demonstrating the validity of rocket description;

3. Demonstrating the validity of dispersion studies.

Appendix A

Procedures for Calculating Wind Weighting Functions and the Impact Displacement Due to Coriolis Force

The Lewis Method

The Lewis method for calculating wind weighting functions was developed by J. V. Lewis* in 1949, and has been widely used. The only rocket data necessary are: thrust vs. time; mass vs. time; and drag coefficient vs. Mach no. The computations are easily set up and rapidly computed.

The Lewis wind weighting function program computes, for each value of launch angle, a table of the impact displacement caused by a unit wind up to altitude z as a function of z. The program uses a perturbation technique, assuming:

1. The rocket body is at all times aligned with the velocity vector of the rocket relative to the air mass. This assumption is called the "particle assumption," the "zero angle of attack assumption," or the "infinite stability assumption";

2. The only forces acting on the rocket are thrust, drag, and gravity;

3. The values for altitude vs. time and vertical velocity vs. time are the same for a trajectory with wind acting on the rocket as for a trajectory with no-wind.

The assumptions of the Lewis method are not accurate when applied to rockets with low velocity immediately after launch. The Lewis method should not be used if sufficient rocket data are available to compute rigid body trajectories.

The Two-Dimensional Rigid Body Program

The two-dimensional rigid body program is not a perturbation method, but requires a complete computation of a rocket trajectory from launch to impact for each change of conditions. It includes equations for the following forces and moments acting on the rocket

1. Gravitational force;

2. Thrust force, acting along the longitudinal axis of the rocket;

3. Drag force, acting in the direction of the relative wind velocity vector, which is the velocity vector of the rocket relative to the moving air;

*Lewis, J. V., "The Effect of Wind and Rotation of the Earth on Unguided Rockets," Ballistic Res. Laboratories Rept. 685, March 1949, Aberdeen Proving Ground, Aberdeen, Maryland.

4. Lift force, acting normal to the drag force, proportional to the sine of the angle of attack;
5. Aerodynamic restoring moment produced by the resultant of the lift and drag forces, acting at the center of pressure;
6. Aerodynamic damping moment;
7. Moment due to jet damping.

Use of the rigid body trajectory program requires the following rocket data:

1. Thrust vs. time;
2. Mass vs. time;
3. Moment of inertia vs. time;
4. Center of gravity vs. time;
5. Center of pressure vs. Mach no.;
6. Drag coefficient vs. Mach no.;
7. Lift coefficient slope vs. Mach no.;
8. Center of pressure for aerodynamic damping vs. Mach no.

The table of $R(\theta)$ is obtained by computing a trajectory from launch to impact with no wind acting on the rocket, for each value of θ, and tabulating range-to-impact vs. θ for each trajectory.

The table of $\delta(\theta)$ is obtained by:

1. Computing a trajectory from launch to impact with a unit wind acting on the rocket from ground level to 100,000 ft, for each value of θ;
2. Taking the difference between each range-to-impact value obtained in the preceding step (1), and the corresponding no-wind range-to-impact value obtained above for the same θ, and tabulating versus θ. In equation form:

$$\delta(\theta) = R(\theta, \text{ unit wind to 100,000 ft }) - R(\theta, \text{ no-wind }).$$

The table of $f(z)$ vs. z is obtained by selecting a standard value of θ to be used for $f(z)$ computation, and then:

1. For each z entry in the $f(z)$ table, computing a trajectory from launch to impact, with a unit wind acting on the rocket from ground level to altitude z and no wind above altitude z. The impact range so obtained is symbolized by $R(\theta$, unit wind to $z)$.
2. Computing the impact displacement due to unit wind to altitude z by the equation

$$\Delta R(\theta, z) = R(\theta, \text{ unit wind to } z) - R(\theta, \text{no-wind}).$$

3. Computing $f(z)$ as the ratio between impact displacement due to unit wind to altitude z and displacement due to unit wind to 100,000 ft,

$$f(z) = \frac{\Delta R(\theta, z)}{\delta(\theta)} .$$

The values of impact displacement due to wind are separated into two components:

1. Those due to the weathercock effect, which turns the rocket into the wind when the rocket motor is operating. This is generally the largest component of the wind effect.
2. Those due to the drift effect, which moves the rocket in the direction of the wind's force during both burning and coast phases. This is in the opposite direction to the weathercock effect.

These two effects are combined in the functions $\delta(\theta)$ and $f(z)$. The $f(z)$ function has a positive slope when the weathercock effect is predominant, and a negative slope when the drift effect is predominant.

The Daw Wind Weighting Procedure

The Daw wind weighting procedure uses a method of computing perturbations to a standard trajectory, with rocket response lag due to inertia included in the perturbation equations. The forces and moments listed in the previous discussion of the two-dimensional rigid body program are included in the equations of perturbations, and the data listed under its program are necessary for computation.

The Daw method relies on an assumption that perturbations computed for a vertical trajectory are also valid for trajectories which are near vertical. The method outlined in the rigid body discussion does not use this assumption. For this reason the rigid body method is preferred to the Daw method.*

Impact Displacement Due to Coriolis Force

The impact displacement due to Coriolis force is computed separately from the wind weighting functions. Two programs are used for this computation:

1. The two-dimensional particle trajectory program, with gravitation computed as a force varying inversely as the square of the distance from the earth's center;
2. The three-dimensional particle trajectory program with an ellipsoidal, rotating earth, having accurate equations for:
   a. The earth's surface, described as an ellipsoid of revolution;

*Daw, H. A., "A Wind Weighting Theory for Sounding Rockets Derivable from the Rocket Equations of Motion," New Mexico State University, November 5, 1958.

b. The earth's gravitational field, described by an expansion in zonal harmonics up to the sixth harmonic;

c. The Coriolis and centrifugal forces.

The displacement of impact due to Coriolis force is assumed to be constant for launch angles near vertical. The displacement vector is obtained by taking the difference between impacts derived from the two programs.*

*Guard, K., "The Trajectory Simulation Programs of the Physical Science Laboratory of New Mexico State University," New Mexico State University, March 30, 1962.

Appendix B

Computer Procedure for Combining Ballistic Wind Profiles from More Than One Source

The combining of ballistic wind profiles from several sources has certain operational difficulties in regard to programming a digital computer. The adopted method differs from that given in the section on "Ballistic Wind Evaluation," but provides the same result. The procedure follows.

Define

$$\{z_i, W_i\}^{(1)}, \text{ where } i = 1, 2, 3, \cdots k, \cdots, n,$$

to be an internally stored table of the ballistic wind profile conforming to

$$W_i = \sum_i \Delta f_i w_i ,$$

and

$$\{z_j, W_j\}^{(2)}, \text{ where } j = 1, 2, 3, \cdots, m,$$

to be a similar set which is to be combined with the former table to provide a new profile for the internally stored table:

$$\{z_i, W_i\}^{(1)}, \text{ where } i = 1, 2, 3, \cdots, n,$$

which is comprised of the most recent observations of the two sets.

Starting with $i = j = 1$, consider two cases:

Case A:

If $z_i^{(1)} \leq z_j^{(2)}$, set:

$$W_i^{(1)} \rightarrow W_t^{(1)},$$

$$z_i^{(1)} \rightarrow z_t^{(1)},$$

$$W_j^{(2)} \rightarrow W_i^{(1)},$$

$$z_j^{(2)} \rightarrow z_i^{(1)};$$

index:

$$j + 1 \rightarrow j$$

$$i + 1 \rightarrow i \; .$$

Case B:

If $z_i^{(1)} > z_j^{(2)}$, index: $j + 1 \rightarrow j$.

The preceding criterion for replacing values of the i set is repeated until $j > m$.

Two cases then exist:

Case 1:

If $z_m \geq z_n$, the entire i set has been replaced by the j set and the total ballistic wind is equal to $W_m^{(2)}$.

Case 2:

If $z_m < z_{n-1}$, the j set has replaced only part of the i set (say k elements of the i set have been replaced).

The replaced values of $z_k^{(1)}$ and $W_k^{(1)}$ have been preserved in $z_t^{(1)}$ and $W_t^{(1)}$, and $z_k^{(2)}$ and $W_k^{(2)}$ are in the internal table. Since $z_t^{(1)} \leq z_k^{(2)} < z_{k+1}^{(1)}$ the expression

$$W_k^{(1)} = \frac{W_{k+1}^{(1)} - W_t^{(1)}}{z_{k+1}^{(1)} - z_t^{(1)}} \left(z_j^{(2)} - z_t^{(1)} \right) + W_t^{(1)} \; ,$$

provides the interpolated value of the partial ballistic wind at $z_k^{(2)}$. All values of $\{z_i, W_i\}^{(1)}$, where $i = k + 1, k + 2, \cdots, n$, are corrected by:

$$W_i^{(1,2)} = W_i^{(1)} + W_k^{(2)} - W_k^{(1)} \; ,$$

and the total ballistic wind is $W_n^{(1,2)}$.

NASA-Langley, 1964 G-251